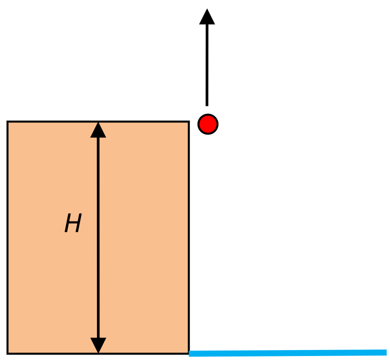
Teacher notes
Topic A

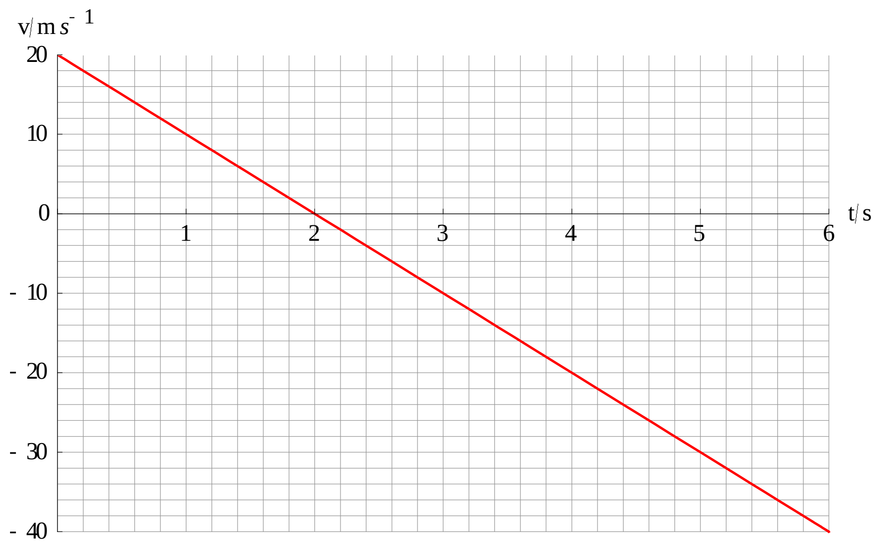
A study of air resistance

In this problem we will take $g = 10\ \text{ms}^{-2}$.

A ball of mass 1.0 kg is projected vertically upwards with an initial speed 20 m s^{-1} from the edge of a cliff at a height H above sea level. The ball reaches a maximum height above the cliff and then falls to the sea.



(a) Ignoring air resistance, the graph of the velocity of the ball versus time is the following:

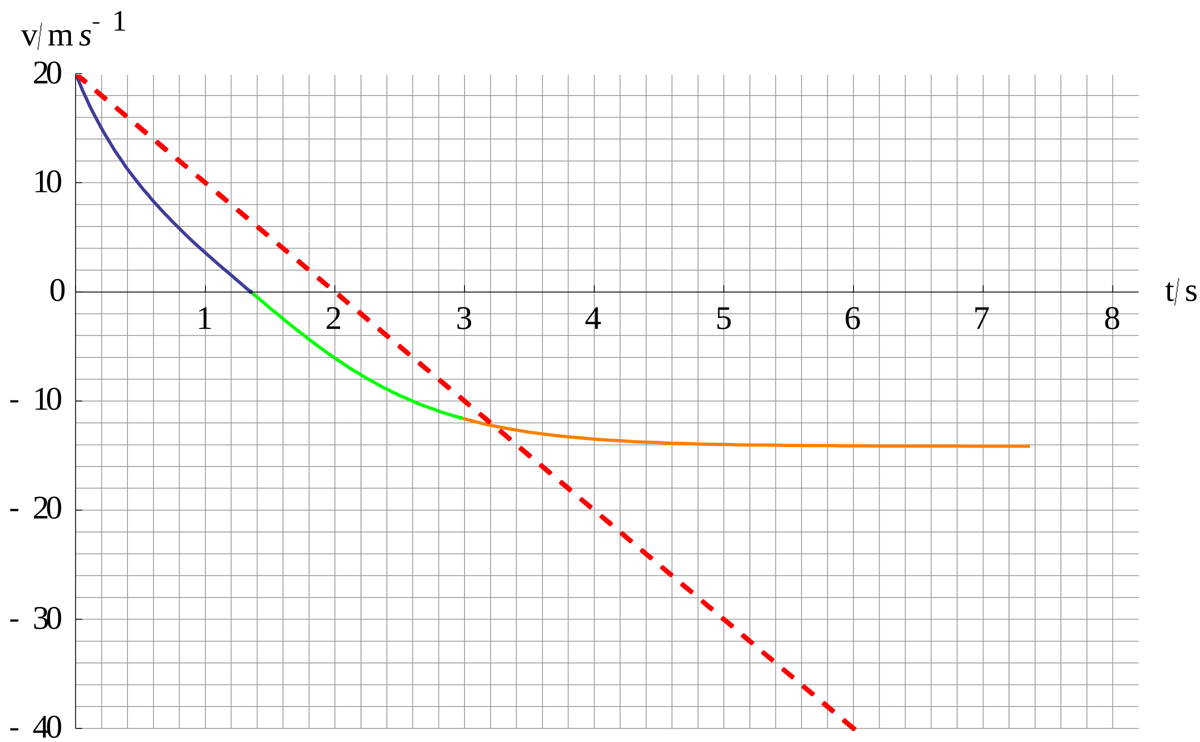


The curve ends when the ball hits the sea.

Use the graph to find

(i) the time taken to reach the maximum height,
(ii) the maximum height of the ball above the cliff,
(iii) the height of the cliff *H*.

(b) In the presence of an air resistance force, the velocity of the ball varies with time as shown. The dashed curve is the original graph with no air resistance. The curve ends when the ball hits the sea.



Different stages in the motion from the cliff to the sea are shown in 3 different colors. Blue: ball from cliff to max height. Green: from max height back to cliff. Orange: from cliff to sea.

(i) State the time to reach the maximum height and the time to descend from the maximum height back to the level of the cliff.
(ii) Explain why these two times are not the same.
(iii) Explain why the speed of the ball as it comes down to the level of the cliff is not the same as the launch speed.
(iv) Estimate the maximum height above the cliff the ball gets to.
(v) Why is the estimate in (b) (iv) different from the answer to (a) (ii)?
(vi) What is the area between the time axis and the orange curve?

(c) The force of air resistance is modeled by $F = kv^2$.

(i) State the unit of k in terms of fundamental units.

(ii) Explain why the ball reaches terminal speed.

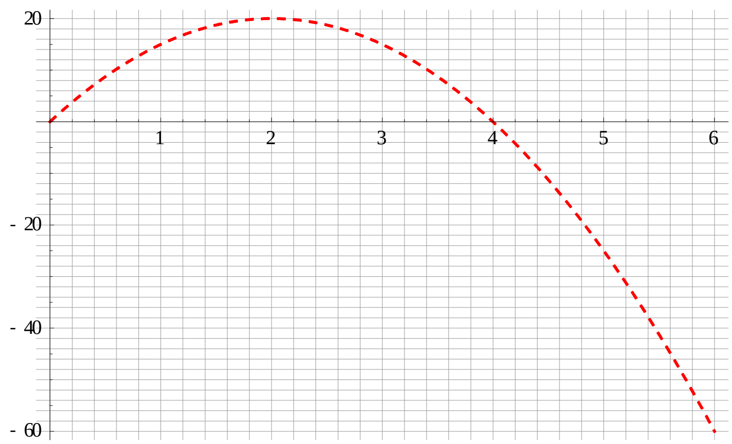
(iii) Calculate k.

(iv) Calculate the acceleration of the ball at the time of launch.

(v) State the acceleration of the ball at the time when the blue and green curves meet commenting on your answer.

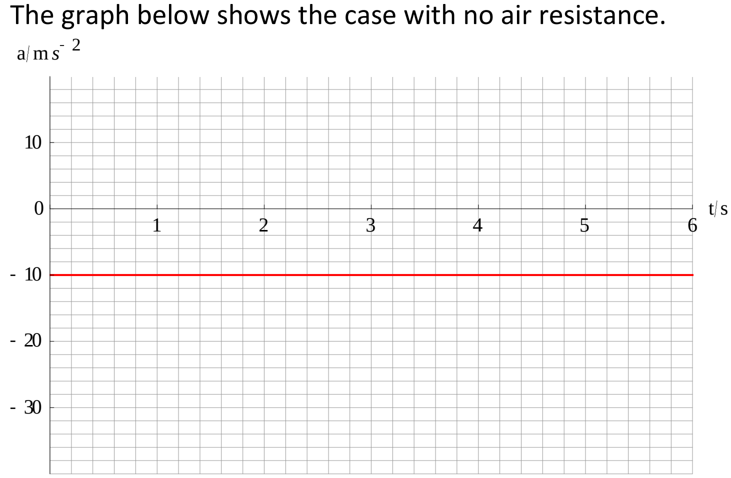
(d) Draw the graph of the variation of the position of the ball with time. A sketch graph only is required.

The graph without air resistance is shown below.



(e) Draw the graph of the variation of the acceleration of the ball with time. A sketch graph only is required.

The graph below shows the case with no air resistance.



Answers

(a)

(i) Where velocity is zero, $t = 2$ s.

(ii) Area from zero to $t = 2$ s gives 20 m.

(iii) Area from $t = 2$ s to $t = 6$ s is -80 m, so total displacement is -60 m and so $H = 60$ m.

(b)

(i) About 1.4 (1.35) s to go up and 1.6 s to come down.

(ii) On the way up both gravity and air resistance oppose motion so acceleration is greater hence time is less.

(iii) Gravitational potential energy goes to thermal energy so kinetic energy back at the cliff is less than that at launch.

(iv) About 10. 5 m, the area under the blue curve.

(v) The acceleration is greater so height reached is smaller ***OR*** thermal energy is produced so mechanical energy at top less than that at launch. At top mechanical energy is just potential and so height has to be less.

(vi) 60 m, the height of the cliff.

(c)

(i) $[k] = \dfrac{\mathrm{N}}{\mathrm{m^2\,s^{-2}}} = \dfrac{\mathrm{kg\,m\,s^{-2}}}{\mathrm{m^2\,s^{-2}}} = \mathrm{kg\ m^{-1}}$.

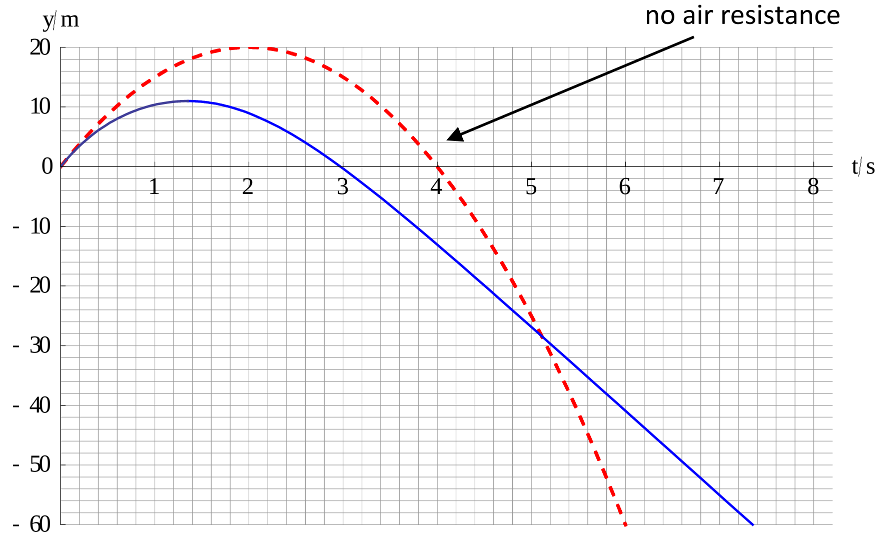
(ii) The force of air resistance increases as speed increases. Eventually it will become equal to the weight and the net force will be zero. So the speed will be constant.

(iii) $kv_{\mathrm{T}}^2 = mg \Rightarrow k = \dfrac{mg}{v_{\mathrm{T}}^2} = \dfrac{1\times 10}{14^2} = 0.05\ \mathrm{kg\,m^{-1}}$.

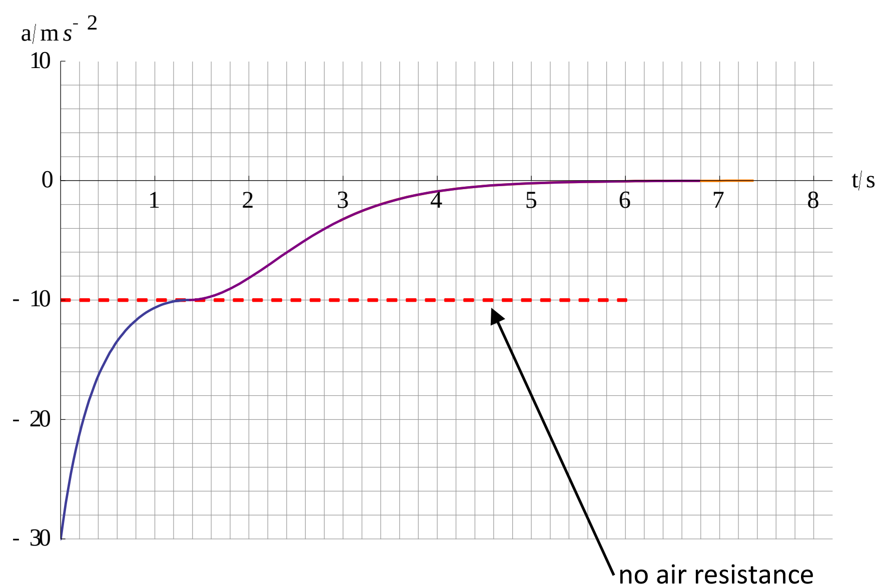
(iv) $ma = -(mg + kv^2) \Rightarrow a = -(g + \dfrac{0.05\times 20^2}{1}) = -30\ \mathrm{m\,s^{-2}}$.

(v) It is $-g$ since there is no air resistance force at this point ($v = 0$). Also obtained by finding the gradient of the tangent to the curve at that point.

(d) You were only expected to get the general shape.



(e) You were only expected to get the general shape. You can be excused for missing the shape near t = 1.4 s; this requires some knowledge of calculus. (See the explanation after the graph if you know calculus.)



Our convention is that the up direction is positive so:

$$a=\begin{cases}-\dfrac{k}{m}v^2-mg & \text{way up}\\ \dfrac{k}{m}v^2-mg & \text{way down}\end{cases}$$

Then

$$\frac{da}{dt}=\begin{cases}-\dfrac{2k}{m}v\dfrac{dv}{dt} & \text{way up}\\ \dfrac{2k}{m}v\dfrac{dv}{dt} & \text{way down}\end{cases}$$

At the top point, $v = 0$ and so $\frac{da}{dt}=0$. A bit before the top point $v>0$, $\frac{dv}{dt}<0$, so $\frac{da}{dt}>0$ and a bit after the top point, $v<0$, $\frac{dv}{dt}<0$, so $\frac{da}{dt}>0$ also. Hence in the acceleration graph the top point (at about t = 1.4 s) is a stationary point of inflection as shown in the graph above.